Effects of interactions and disorder on the compressibility of two-dimensional electron and hole systems

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Abstract

The compressibility \( \chi \) of dilute two-dimensional electron and hole gases in GaAs semiconductor structures has been studied in the ranges of the interaction parameter \( r_s = 1–2.5 \) and \( r_s = 10–30 \) for the electron and hole system, respectively. Nonmonotonic dependence of \( \chi^{-1} \) with an upturn at low carrier densities is observed. Despite the large difference in \( r_s \) the behavior of \( \chi^{-1} \) in both systems can be accurately described by the theory of nonlinear screening of disorder by the carriers.

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1. Introduction

A fundamental property of a conductor is its charge compressibility defined by \( \chi = d\rho/d\mu \), where \( \mu \) is the chemical potential and \( n \) is the carrier density. The dependence of \( \chi \) on \( n \) can reveal important information about both electron–electron interactions and disorder-induced localization. In a pioneering work on a 2DEG [1] it was shown that \( \chi^{-1}(n) \) has a minimum at some \( n = n_m \) and develops an upturn at lower \( n \). The latter was explained in [2] as an effect of disorder that produces depletion regions (DRs), i.e., areas of locally suppressed electron density in the 2D channel. Later, the compressibility was measured in a 2DHG [3] that exhibited, at some low density \( n_c \), the apparent metal–insulator transition (MIT) (the sign change of \( d\rho_{xx}(T)/dT \) with decreasing density [4]). A similar pattern of \( \chi^{-1}(n) \)—a minimum followed by an upturn as \( n \) decreases—was reported. In addition, an intriguing coincidence of \( n_c \) and \( n_m \) was observed. This was interpreted as a thermodynamic signature of a phase transition [3]. One can argue [4] that such a transition could indeed occur in a 2DHG because of a much higher dimensionless strength of Coulomb interaction \( r_s = 1/\sqrt{\pi \alpha_B} \) compared to the 2DEG [1]. (Here \( \alpha_B = 4\pi\epsilon_0\hbar^2/m^*e^2 \) is the effective Bohr radius.) The relation between the upturn of \( \chi^{-1}(n) \) and the nonuniformity of the spatial distribution of electrons due to disorder was studied numerically in Ref. [5].

With the aim of understanding the behavior of the \( \chi \) at low densities we have performed measurements of the compressibility both in a 2DEG with \( r_s < 2.5 \) and a 2DHG with \( r_s = 10–30 \). We show that in both samples the observed \( \chi^{-1}(n) \) dependence can be explained without invoking a quantum phase transition. We find instead a quantitative agreement with a recent work of one of us [6] in which analytical formulae for \( \chi(n) \) applicable in a broad range of \( r_s \) were derived by extending the original theory of Ref. [2].
2. Experiment

The studied 2DEG is a top layer in a double-layer GaAs/AlGaAs heterostructure with the following parameters: a doped layer of thickness 400 Å and concentration $1 \times 10^{18}$ cm$^{-3}$ separated from the 2DEG by a 200 Å spacer; the separation between the two quantum wells of width 200 Å is 200 Å. The 2DHG is part of a single-layer GaAs/AlGaAs heterostructure, with a doped layer of thickness 2000 Å and concentration 0.11 $\times 10^{18}$ cm$^{-3}$, separated from the 2DEG by a 500 Å spacer. The measurements of the compressibility were performed in the temperature range from 30 mK to 10 K at different gate voltages $V_g$. In the double-layer structure, the compressibility of the top layer has been measured using both the direct capacitance technique and penetration-field technique [1]. Fig. 1(a). In the direct method, an AC voltage $\tilde{V}_g = 2$ mV with frequency 1–300 Hz is applied to the gate, and the capacitive current $I_t$ in the top conductive channel (at 90° shift with respect to the AC voltage) is measured. From its value the capacitance $C_t = I_t/2\pi f \tilde{V}_g$ is determined, which can be presented as two capacitors in series: $C_t^{-1} = C_L^{-1} + C_E^{-1}$, where $C_L$ is the geometric contribution due to the voltage drop between the gate and the channel and $C_E = e^2(\partial n/\partial \mu)$ is the electronic (compressibility) contribution. In the penetration-field method, the capacitive current in the bottom layer is measured and the corresponding value of capacitance $C_b = I_b/2\pi f \tilde{V}_g$ is obtained. The value of the capacitive current in this case is determined by the electric field penetrating through the top layer and is proportional to the value $\partial \mu/\partial n$ in this layer [1]. The advantage of this method is that the geometric capacitance does not affect the measurement of $C_E$. By comparing the results of two methods on the double-layer structure we have shown that in the range of low carrier densities they give identical results (see below). In the single-layer 2DHG the direct capacitance measurements were performed to obtain the value of the compressibility. All measurements were done at low enough frequencies so that the resistance of the channel, increasing at low densities, did not affect the measurement of the capacitive current.

It is convenient to analyze the results of measurements in terms of the distance parameters $d_t$ and $d_b$ [1]: $d_{t,b} = (\varepsilon \varepsilon_0/e^2)\partial \mu_{t,b}/\partial n_{t,b}$, which are proportional to the inverse compressibility of the top and bottom layers, respectively. (In single-layer structures only $d_t$ is determined.) Fig. 2 shows the measured dependence $C_t(V_g)$ from which $d_t$ is determined using the relation:

$$d_t = \varepsilon \varepsilon_0 A (C_t^{-1} - C_L^{-1})/(1 + \gamma^{-1}),$$

(1)

where $A$ is the area of the gate and $C_L = \varepsilon \varepsilon_0 A/s_1$ is geometrical capacitance, $s_1$ is the distance between the top channel and the gate, $\gamma^{-1} = s_1/(d_t + s_2)$ is a factor which takes into account the presence of the bottom layer in the double-layer structure, $s_2$ is the distance between the layers. (In a single-layer structure $s_2 \to \infty$ and $\gamma^{-1} = 0$.) In the penetration-field technique [1] $d_t$ is calculated from $C_b(V_g)$ in Fig. 2(a) using Eqs. (7)–(9) of Ref. [1].

3. Analysis

In real 2D systems, a number of effects contribute to the charge compressibility [1]. Our analysis below is based on the relation:

$$d_t = d_B + \Delta d_{ex} + \Delta d_{cor} + \Delta d_{dis} + \Delta d_{sub}.$$  

(2)

The first term in Eq. (2) is due to the single-particle density of states (kinetic energy) of the 2D electrons. The correction $\Delta d_{ex} = -(8\pi^2 n)^{-1/2}$ comes from the exchange interaction. Notably, it has negative sign. Another negative contribution comes from the correlation energy: $\Delta d_{cor} = (\varepsilon \varepsilon_0/e_2^2)(nE_c)/dn^2$. The correlation energy per particle $E_c$ is computed according to the interpolation formula of Ref. [7]. As the density decreases, $\Delta d_{cor}$ becomes
comparable to $\Delta d_{\text{ex}}$ and acting together they cause the sign change of $d_1$ from positive to negative as seen experimentally [1,3,8]. The disorder makes a positive contribution $\Delta d_{\text{dis}}$, which can be responsible for the upturn in the inverse compressibility at low densities [6]:

$$
\Delta d_{\text{dis}} = \frac{s}{4\pi} \frac{3\sqrt{2}}{8\pi \eta} \left[ 0.3 + \eta \frac{0.036 + 0.12\eta + \eta^2 e^{-4\eta^2}}{\sqrt{n_i}} \right],
$$

where $\eta = n s/\sqrt{n_i}$ and $n_i$ is the effective 2D concentration of dopants (an adjustable parameter, see below). Physically, $\Delta d_{\text{dis}}$ arises due to the aforementioned DRs in the conducting channel, i.e. the areas of exponentially small concentration. A DR can be thought of as a region where the screening of disorder by the electron liquid is driven into the extreme nonlinear limit. The upturn of $\chi^{-1}$ originates from the excess energy of the electron system associated with such a nonlinearity.

The last term in Eq. (2), $\Delta d_{\text{sub}}$, accounts for the variation of the subband energy with the carrier concentration. It arises because the electric field at the interface changes with $n$. The expression for $\Delta d_{\text{sub}}$ is different for the two structures used because they have different types of confinement in the third direction (heterostructure and a quantum well). For a single-layer structure we use the standard formula [9]:

$$
\Delta d_{\text{sub}} = \frac{e\varepsilon_0}{\varepsilon} \frac{dE_0}{dn} = \frac{55}{32} \left( \frac{33\pi}{2} \frac{n}{a_B^2} \right)^{-1/3},
$$

where $E_0$ is the energy of the lowest subband. For a double-layer structure we compute $\Delta d_{\text{sub}}$ using the infinite square-well approximation for the confining potential. The result depends on the type of measurement. For the direct capacitance method we obtain:

$$
\Delta d_{\text{sub}} = \frac{8\pi^2 + 15}{24\pi^2} w_t - \frac{1}{2\pi^4} \left( 5 - \frac{81}{8\pi^2} - \frac{4\pi^2}{15} \right) \frac{mn^4}{a_B},
$$

where $w_t$ is the thickness of the quantum well. The overall contribution $\Delta d_{\text{sub}}$ is positive in this case. In contrast, in the penetration-field technique $\Delta d_{\text{sub}}$ is negative,

$$
\Delta d_{\text{sub}} = -\frac{4\pi^2 - 15}{24\pi^2} w_t + \frac{1}{2\pi^4} \left( \frac{81}{8\pi^2} - \frac{4\pi^2}{15} \right) \frac{mn^4}{a_B},
$$

in a close agreement with the interpolation formula found in Ref. [1] from numerical simulations. To compare our experimental results with the above theoretical predictions, we present them in the form of $d^* = d_1 - \Delta d_{\text{sub}}$ as a function of the carrier density. The carrier density at a given gate voltage was found both from measurements of the Hall effect and the integral of the capacitance $C(V_{g})$ at $f \to 0$.

4. Results and discussion

Fig. 3 shows the results for the 2DEG obtained by both direct capacitance and penetration-field methods. It is seen that the two methods agree well with each other. The dotted line shows the best fit to the theoretically predicted $d^*$ [Eqs. (2)–(6)] using one adjustable parameter, $n_i = 1.2 \times 10^{11} \text{cm}^{-2}$. Fig. 4 shows the results for the 2DHG together with theory using the concentration of acceptors $n_i = 1.0 \times 10^{11} \text{cm}^{-2}$. Good overall agreement with the theory is obtained for both 2DEG and 2DHG structures.

An interesting point to note is that the above values of $n_i$ are lower than expected from the growth parameters of the structures. Indeed, for the 2DEG the best fit was obtained with $n_i = 1.2 \times 10^{11} \text{cm}^{-2}$, while the expected is $9 \times 10^{11} \text{cm}^{-2}$ (taking into account the thickness of the doped layer and the presence of the gate). For the 2DHG $n_i = 1.0 \times 10^{11} \text{cm}^{-2}$ while the expected one is $5.6 \times 10^{11} \text{cm}^{-2}$. The reason for such a discrepancy can be due to the fact that the theory [6] assumes uncorrelated positions of ionized donors and acceptors, while in reality certain correlations among them exist [10]. This effectively
decreases the disorder and as a result the value of $n_i$ [2]. In our experiment, we have detected a shift in $\chi^{-1}$ from one cool-down to another. This supports the suggestion that correlation in the impurity positions can exist, since those depend on the experimental conditions [10].

Finally, we comment on the position of the minimum in $\chi^{-1}$. In order to study the relation between $n_m$ and $n_c$ (the apparent MIT), the compressibility needs to be analyzed in conjunction with the results of transport measurements. This comparison will be reported elsewhere.

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References