QUANTUM HALL LIQUID CRYSTALS

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The stripe phase of a two-dimensional electron system in a weak magnetic field bears a close analogy to liquid crystals. However, reduced dimensionality and unusual dynamics give rise to important differences. At finite temperature they cause divergent fluctuations and nonperturbative renormalization of hydrodynamic parameters. Such effects can be verified in microwave experiments. At low temperatures the physics is dominated by quantum fluctuations. When they are large, the transition to a novel quantum nematic phase may occur, driven by quantum proliferation of dislocations. It will be signaled by an additional low-frequency resonance in the microwave response.

1 Introduction

Historically, most of the research in the area of the quantum Hall effect has been focused on the case of very strong magnetic fields \( B \) where all the electrons reside at the lowest Landau level (LL). Recently, it has been discovered that moderate and weak magnetic fields, i.e., high LLs, is also a realm of very interesting physics.\(^1\) A partially filled high LL undergoes a charge-density wave (CDW) transition below a temperature \( T_{c}^{\text{mf}} \sim 0.06e^{2}/\kappa R_{c} \) where \( R_{c} \propto 1/B \) is the classical cyclotron radius and \( \kappa \) is the bare dielectric constant. Near half-filling, \( \nu \sim 2N + \frac{1}{2} \), the resultant CDW is a unidirectional, i.e., the stripe phase. At other filling fractions, the CDW has a symmetry of the triangular lattice and is called the bubble phase. At low temperatures the system becomes divided into depletion regions where the local filling fraction is equal to \( 2N \), and stripe- or bubble-shaped domains with the local filling fraction \( 2N + 1 \). The CDW periodicity is set by the wavevector\(^1\) \( q_{\ast} \approx 2.4/R_{c} \). In the quasiclassical limit of large LL indices \( N \) the CDW is well described by the mean-field theory.\(^1,2\) At moderate \( N \) there are sizeable fluctuations around the mean-field solution, which may lead to a new physics described below. At such \( N \) the CDW phases compete with Laughlin liquids and other fractional quantum Hall (FQH) states. A combination of analytic and numerical tools\(^1,3,4,5\) suggests that the FQH states lose to the CDW at \( N \geq 2 \). The existence of the stripe phase as a physical reality was evidenced by a conspicuous magnetoresistance anisotropy observed near half-integral fractions of high LLs.\(^6,7\) This stimulated a considerable amount of theoretical work devoted to the stripes. It led to the understanding that the “stripes” may appear in several distinct
forms: an anisotropic crystal, a smectic, and a nematic (Fig. 1). These phases succeed each other in the order listed as the magnitude of either quantum or thermal fluctuations increases. The general structure of a phase diagram that includes these novel phases was discussed in the important paper of Fradkin and Kivelson\(^8\) (for \(T = 0\)). The most intriguing are the phases which bear the liquid crystal names: the smectic and the nematic.

The smectic is a liquid with the 1D periodicity, i.e., a state where the translational symmetry broken only in one spatial direction.\(^9\) An example of such a state is the original Hartree-Fock stripe solution\(^1\) although a stable quantum Hall smectic must have a certain amount of quantum fluctuations around the mean-field state.\(^10,11\) The necessary condition for the smectic order is the continuity of the stripes. If the stripes are allowed to rupture, the dislocations are created. They destroy the 1D positional order and convert the smectic into a nematic.\(^12\)

By definition, the nematic is an anisotropic liquid.\(^9\) There is no long-range positional order. As for the orientational order, it is long-range at \(T = 0\) and quasi-long-range (power-law correlations) at finite \(T\). The nematic is riddled with dynamic dislocations.

It is often the case that the low-frequency long-wavelength physics of the system is governed by an effective theory involving a relatively small number of dynamical variables. In the remaining sections I will discuss such type of theories for the quantum Hall liquid crystals.

\section*{2 Smectic state}

The collective variables in the smectic are (i) the deviations \(u(x, y)\) of the stripes from their equilibrium positions and (ii) long-wavelength density fluctuations \(n\) about the average value \(n_0\). The latter fluctuations may originate, e.g., from width fluctuations of the stripes. Let us assume that the stripes are aligned in the \(\hat{y}\)-direction, then the symmetry considerations fix the effective
Hamiltonian for $u$ and $n$ to be\textsuperscript{9,13}
\[ H = \frac{Y}{2} \left[ \partial_x u - \frac{1}{2} (\nabla u)^2 \right]^2 + \frac{K}{2} (\partial_y u)^2 + \frac{1}{2} n V n, \]
(1)
where $Y$ and $K$ are the phenomenological compression and the bending elastic moduli, and $V(r) = e^2/k\tau$ should be understood as the integral operator. The dynamics of the smectic is dominated by the Lorentz force and is governed by the Largangean\textsuperscript{14}
\[ \mathcal{L} = p \partial_t u - H, \quad \partial_y p = -m \omega_c (n + n_0 \partial_x u) \]
(2)
where $m$ is the electron mass and $\omega_c = eB/mc$ is the cyclotron frequency.

It is natural to start with the harmonic approximation where one replaces the first term in $H$ simply by $(Y/2)(\partial_x u)^2$. Solving the equations of motion for $n$ and $u$ we obtain the dispersion relation for the phonon-like vibrations of the stripes (referred to as magnetophons in what follows).\textsuperscript{13}
\[ \omega(q) = \frac{\omega_p(q)}{\omega_c} \frac{q_y}{q} \left[ \frac{Y q_x^2 + K q_y^4}{mn_0} \right]^{1/2}. \]
(3)
Here $\omega_p(q) = [n_0 V(q)q_x^2/m]^{1/2}$ is the plasma frequency and $\theta = \arctan(q_y/q_x)$ is the angle between the propagation direction and the $\hat{x}$-axis. For Coulomb interactions $\omega_p(q) \propto \sqrt{q}$. Unless propagate nearly parallel to the stripes, $\omega(q)$ is proportional to $\sin 2\theta q^{3/2}$. One immediate consequence of this dispersion is that the largest velocity of propagation for the magnetophons with a given $q$ is achieved when $\theta = 45^\circ$.

At any finite $T$, harmonic fluctuations of the stripe positions become larger than the interstripe separation at distances exceeding $\xi_y \sim \sqrt{Y K}/k_B T q_*$. and $\xi_x = (Y/K)^{1/2} \xi_y^2$ along the $\hat{y}$- and $\hat{x}$-directions, respectively. The stripe positions are also disordered by the dislocations. The dislocations in a 2D smectic have a finite energy $E_D \sim K$. At $k_B T \ll E_D$ the density of thermally excited dislocations is of the order of\textsuperscript{12} $\exp(-E_D/k_B T)$ and the average distance between dislocations is $\xi_D \sim q_*^{-1} \exp(2k_B T/E_D)$. At low temperatures $\xi_x, \xi_y \ll \xi_D$; therefore, the following interesting situation emerges (Fig. 2). On the lengthscales smaller than $\xi_y$ (or $\xi_x$, whichever appropriate) the system behaves like a usual smectic where Eqs. (1–3) apply. On the lengthscales exceeding $\xi_D$ it behaves\textsuperscript{a} like a nematic.\textsuperscript{12} In between the system is a smectic but with very unusual properties. It is topologically

\textsuperscript{a} In a more precise treatment,\textsuperscript{15} the lengthscales $\xi_{Dx} \propto \xi_D^{6/5}$ and $\xi_{Dy} \propto \xi_D^{4/5}$ are introduced such that $\xi_{Dx}, \xi_{Dy} = \xi_D^2$. 


ordered (no dislocations) but possesses enormous fluctuations. In these circumstances the harmonic elastic theory becomes inadequate and anharmonic terms must be treated carefully. The calculations$^{15,13}$ show that the anharmonisms cause power-law dependence of the parameters of the effective theory on the wavevector $q$, e.g.,

$$Y \sim Y_0 (\xi_x q_x)^{1/3}, \quad K \sim K_0 (\xi_x q_x)^{-1/3},$$  \hspace{1cm} (4)

for $q_x \ll \xi_x^{-1}$ and $q_y \ll \xi_y^{-1}(q_x \xi_x)^{2/3}$. The scaling behavior (4) breaks down above the lengthscale $\xi_D$ where the crossover to the thermodynamic limit of the nematic behavior commences.

The scaling shows up not only in the static properties such as $Y$ and $K$ but also in the dynamics. For example, the spectrum of the magnetophonon modes changes to$^{13}$

$$\omega(q) \sim \sin \theta \cos^{7/6} \theta (\xi_x q)^{5/3} \omega_p(\xi_x^{-1}) \sqrt{\frac{Y_0}{\omega_c \xi_x}} \sqrt{\frac{mn_0}{\omega_c \xi_x}}.$$  \hspace{1cm} (5)

Compared to the predictions of the harmonic theory, Eq. (3), the $q^{3/2}$ dispersion changes to $q^{5/3}$. Also, the maximum propagation velocity is achieved for the angle $\theta \approx 53^\circ$ instead of $\theta = 45^\circ$. These modifications, which take place at long wavelengths, are mainly due to the renormalization of $Y$ in the static limit and can be obtained by combining Eqs. (3) and (4). Less obvious dynamical effects peculiar to the quantum Hall smectics include a novel dynamical scaling of $Y$ and $K$ as a function of frequency and a specific $q$-dependence of the magnetophonon damping.$^{13}$
3 Nematic

The collective degree of freedom associated with the nematic ordering is the angle $\phi(r,t)$ between the local normal to the stripes $N$ and the $x$-axis orientation. The effective Hamiltonian for $N$ is dictated by symmetry to be

$$H_N = \frac{K_1}{2} (\nabla N)^2 + \frac{K_3}{2} |\nabla \times N|^2.$$  \hspace{1cm} (6)

The coefficients $K_1$ and $K_3$ are termed the splay and the bend Frank constants\textsuperscript{9}. Note that in the smectic phase $\phi = -\partial_y u$. This entails the relation $K_3 \simeq K$ between the parameters of the nematic and its parent smectic. On the other hand, the value of $K_1$ is expected to be determined largely by the properties of the dislocations\textsuperscript{12}.

Another obvious degree of freedom in the nematic are the density fluctuations $n(r,t)$. A peculiar fact is that in the static limit $n$ is totally decoupled from $N$, and so it does not enter Eq. (6). Since the nematic is a rather weak form of ordering, the question about extra low-energy degrees of freedom or additional quasiparticles is nevertheless relevant. I believe that different types of quantum Hall nematics are possible in nature. In the simplest case scenario $N$ and $n$ are the only low-energy degrees of freedom. This type of state has been studied by Balents\textsuperscript{16} and recently by the present author\textsuperscript{14}. It was essentially postulated that the effective Largangean takes the form

$$\mathcal{L} = \frac{1}{2} \gamma^{-1} (\partial_t N)^2 - H.$$ \hspace{1cm} (7)

(As hinted above, the full expression contains also couplings between $\partial_t N$ and mass currents but they become vanishingly small in the long-wavelength limit). The collective excitations are charge-neutral fluctuations of the director. They have a linear dispersion,

$$\omega(q) = q \sqrt{K_1 \gamma \cos^2 \theta + K_3 \gamma \sin^2 \theta}.$$ \hspace{1cm} (8)

One interesting question is the nature of the zero-temperature smectic-nematic transition. It is likely to be dislocation-mediated, which can be studied\textsuperscript{14} combining classical\textsuperscript{12,17} and quantum\textsuperscript{18} theories of topological disordering. One prediction\textsuperscript{14} of this scenario is the existence of a second gapped excitation branch in the nematic. This mode is a descendant of the magnetophonon mode of the parent smectic.

Very recently, Radzihovsky and Dorsey\textsuperscript{19} formulated a qualitatively different theory of the quantum Hall nematics, whose predictions disagree with our Eqs. (7) and (8). To resolve some of the controversy it is imperative to
bring the discussion from the level of effective theory to the level of quantitative calculations. One promising direction is to investigate some concrete trial wavefunctions of quantum nematics.\textsuperscript{20,21} It is worth mentioning that the quantum phase transition(s) from the smectic to an isotropic state may also occur directly, without the intermediate nematic phase.\textsuperscript{22}

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References