Comment on “Analytic Structure of One-Dimensional Localization Theory: Reexamining Mott’s Law”

The low-frequency conductivity \(\sigma(\omega)\) of a disordered Fermi gas in one spatial dimension is governed by the Mott-Berezinskii law [1–3]

\[
\sigma(\omega) = \sigma_0 (2\xi(3)e - e^2[\ln^2 e + (2C - 3)\ln e - c]),
\]

(1)

Here \(\sigma_0\) is the Drude conductivity, \(e = -2i\omega\tau\) is a small parameter in the problem, \(\tau\) is the elastic scattering time, \(C\) is the Euler constant, and \(c = O(1)\) is another constant. In a recent Letter [4] Gogolin claimed that the expansion in powers of \(\ln e\) inside the square brackets in Eq. (1) starts with the \(\ln^3 e\) term, challenging our basic ideas about disordered systems [1], as well as previous analytical [2,5,6] and numerical [7] work. Below we reinstate the validity of the Mott-Berezinskii law by pointing out two calculational errors in Gogolin’s paper. We also present numerical results for \(\sigma\), which fully agree with Eq. (1).

Gogolin expresses \(\sigma(\omega)\) as a sum over certain matrix elements \(f_n\) of the eigenfunctions of Berezinskii’s second-order differential equation:

\[
\sigma(\omega) = \sum_{n=1}^{\infty} f_n, \quad f_n = \frac{\pi^2\sigma_0 e}{\cosh^2\pi k_n} \frac{B_{ne}^2}{2k_n}. \tag{2}
\]

The wave numbers \(k_n\) are to be determined from the equation \(g(k_n) = 2\pi n\), where

\[
g(k) = 2kL - i\ln\left[\frac{\Gamma^2(1 + ik)\Gamma(\frac{1}{2} - i k)}{\Gamma^2(1 - ik)\Gamma(\frac{1}{2} + i k)}\right], \tag{3}
\]

and \(L = \ln(16/\varepsilon)\). The Poisson summation formula brings Eq. (2) to the form

\[
\sigma(\omega) = \frac{1}{4\pi} \sum_{p=-\infty}^{\infty} \int dk f(k)e^{ipg(k)}g'(k). \tag{4}
\]

The third power of \(\ln e\) was obtained in Ref. [4] because \(g(k) = 2kL\) was used instead of the full expression (3). This is the crucial mistake in that paper. To get the correct coefficient in front of the subleading term in Eq. (1), one more mistake has to be rectified. Namely, one has to use the following result for \(B_{ne}^2\):

\[
\frac{1}{B_{ne}^2} = \frac{\pi L}{2k_n\sinh\pi k_n} - \frac{\pi}{4} \frac{\Gamma(\frac{1}{2} - ik_n/\pi)\Gamma(\frac{1}{2} + ik_n/\pi)}{\Gamma(\frac{3}{4} - \frac{ik_n}{\pi})\Gamma(\frac{3}{4} + \frac{ik_n}{\pi})}. \tag{5}
\]

The second term has to be retained because it diverges at the point \(k_n = i/2\) where the integrand in Eq. (4) has a pole. Once we substitute more accurate expressions (3) and (5) into Eqs. (2) and (4), Eq. (1) is fully recovered.

As a final check, we solved Berezinskii’s recursive equations [2,4] numerically for small real \(\varepsilon\), slightly refining the algorithm described in Ref. [7]. The solid line in Fig. 1 is the best fit of \(\sigma(\varepsilon)\) to the form \(\sigma_B(\varepsilon) = 8\xi(3)e/\pi - \beta e^2(\ln^2 e - \gamma \ln e + \delta) (\sigma_0 = 4/\pi \text{ in our system of units}, \text{which is provided by } \beta = 1.272(5), \gamma = 1.85(1), \text{and } \delta = 2.11(1), \text{in excellent agreement with } \beta = 4/\pi = 1.2732 \ldots, \gamma = 3 - 2C = 1.8456 \ldots, \text{predicted by Eq. (1)}. \text{In the inset, we plot } [\sigma(\varepsilon) - \sigma_B]/e^2 \text{ using the latter values of } \beta \text{ and } \gamma \text{ and } \delta = 0. \text{This quantity tends to a constant at small } \varepsilon \text{ in accordance with Eq. (1)}. \text{Z.W. is supported by DOE Grant No. DE-FG02-99ER45747.}

Note added.—Similar numerical results have been independently obtained in Ref. [8].

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[3] The \(\ln^3 e\) factor in Eq. (1) was actually pointed out to Mott by B. I. Halperin; see Ref. [1].