Cyclotron Resonance in a Two-Dimensional Electron Gas with Long-Range Randomness

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We show that the cyclotron resonance in a two-dimensional electron gas has nontrivial properties if the correlation length of the disorder is larger than the Fermi wavelength: (a) The line shape assumes three different forms in strong, intermediate, and weak magnetic fields. (b) The linewidth collapses at the transition from the intermediate to the weak fields via the motional narrowing mechanism brought about by a dramatic enhancement of the localization length. [S0031-9007(98)06136-5]

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The theoretical study of the CR in the 2DEG was initiated by Ando [1] almost two decades ago and then continued by others [2,3]. Surprisingly, only the short-range disorder case \( k_F d \ll 1 \) received its essentially complete description. Here \( d \) is the correlation length of the random potential acting on the electrons and \( k_F \) is the Fermi wave vector in zero magnetic field. The effect of a short-range random potential is described by a single quantity, the transport (or momentum relaxation) time \( \tau \). Nevertheless, the classical formula

\[
\text{Re} \sigma_+(\omega) = \frac{\sigma_0}{1 + (\omega - \omega_c)^2 \tau^2} \tag{1}
\]

(\( \sigma_0 \) is the zero field conductivity), which gives the Lorentzian peak with the half width at half maximum (HWHM) of \( \tau^{-1} \), applies only in the uninteresting case \( \omega_c \tau \ll 1 \). In the other limit \( (\omega_c \tau \gg 1) \) the CR line shape is non-Lorentzian and has a much larger width [1]

\[
\Delta \omega_{1/2} = 0.73 \sqrt{\omega_c \tau} \tag{2}
\]

due to the formation of discrete Landau levels. This behavior of \( \Delta \omega_{1/2} \) is illustrated by the thin line in Fig. 1.

Here and below \( \Delta \omega_{1/2} \) is the median width defined by

\[
\int_0^{\Delta \omega_{1/2}} d\omega \text{ Re} \sigma_+(\omega + \omega_c) = \frac{1}{2} \int_0^\infty d\omega \times \text{ Re} \sigma_+(\omega + \omega_c).
\]

Using the median width instead of the conventional HWHM is more adequate because the CR line shape can be rather intricate for a long-range random potential.

For simplicity, we will consider a model of a Gaussian random potential \( U(x, y) \) whose correlator decays sufficiently fast at distances larger than \( d \) and does not possess any other characteristic scales besides \( d \). The root mean square (rms) amplitude of \( U \) will be denoted by \( W \). Until the very end we will assume that \( k_F d \gg (E_F/W)^{2/3} \), where \( E_F \) is the Fermi energy. As one can see from Fig. 1, the dependence of \( \Delta \omega_{1/2} \) on the magnetic field is nonmonotonic. Even more remarkable, \( \Delta \omega_{1/2} \) exhibits a rapid collapse to its classical value of \( \tau^{-1} \) in the vicinity of the point \( \omega_c \tau \sim (E_F/W)^{2/3} \gg 1 \). The derivation of these results is based on the picture of the “classical localization” [4,5], which we will now discuss.

If the random potential \( U \) is smooth and the time scale on which we study the motion of an electron is not too long, then it can be described classically, as a motion of a single particle with energy \( E_F \). We neglect the interaction.) If the magnetic field is not too low, the motion can be decomposed into a fast cyclotron gyration

\[
\Delta \omega_{1/2}
\]

FIG. 1. Dependence of the CR linewidth on \( (\omega_c \tau)^{-1} \). Thick line: our results for the long-range potential with given \( W/E_F \) and \( \tau \) [for the case \( k_F d \gg (E_F/W)^{2/3} \)]. Labels (1), (7), and (8) correspond to the equation numbers. Thin line: short-range potential with the same \( \tau \). Dash-dotted line: the width \( v \) of an additional structure near the very resonance (see text).
and a slow drift of the guiding center $\rho = (\rho_x, \rho_y)$ of the cyclotron orbit. For magnetic field in the negative $z$ direction, $\rho_x = x + (v_y / \omega_c)$ and $\rho_y = y - (v_x / \omega_c)$, where $x$ and $y$ are the coordinates of the electron and $v = -v_F (\sin \theta, \cos \theta)$ is its velocity. We will call the magnetic field strong if the cyclotron radius $R_c = v_F / \omega_c$ is smaller than $d$. In such strong fields the guiding center drifts along a level line $U(\rho) = \text{const}$ of the random potential, which is typically a closed loop of size $d$. The drift picture of the electron motion was used in the percolation theory of the quantum Hall effect [4].

Recently it has been realized [5,6] that the drift approximation is also valid in the intermediate field regime $1 < R_c / d < (E_F / W)^{2/3}$ [the same as $(E_F / W)^{1/3} < \omega_c \tau < (E_F / W)^2$] because $\tau \sim (d / v_F) (E_F / W)^2$. The point is that in this regime the cyclotron gyration is still sufficiently fast, so that the guiding center remains practically “frozen” during one cyclotron period. Therefore, the guiding center motion is determined by $U_0$, the random potential averaged over the cyclotron orbit,

$$U_0(\rho, R_c) = \frac{1}{2\pi} \int_0^{2\pi} \frac{d\theta}{2\pi} U(\rho_x + R_c \cos \theta, \rho_y + R_c \sin \theta).$$

The guiding center is still bound to one of the level lines, but those are the level lines of $U_0$, not $U$ [5–7]. The rms amplitude $W_0$ of the potential $U_0$ is by a factor of $\sqrt{d / R_c}$ smaller than $W$ due to the averaging, but the correlation length is the same; therefore, a typical level line of $U_0$ is still a loop of size $d$. The frequency $\omega_d$ of the guiding center motion along such a loop (the drift frequency) is given by $W_0 / m \omega_c \bar{d}^2$, which can be cast into the form

$$\omega_d = (R_c / d) \sqrt{\omega_c / \tau}, \quad R_c \gg d. \quad (3)$$

Obviously, the electrons on the periodic orbits are (classically) localized and do not participate in the dc transport. However, a more accurate analysis [5] reveals that a very small fraction of the order of $e^{- (\omega_d / \omega_c)^{3/2}}$ of the trajectories remains delocalized. Such trajectories form a stochastic web in the vicinity of the percolation contour. The web rapidly grows with decreasing magnetic field and turns into a stochastic sea at

$$R_c / d = (E_F / W)^{2/3}, \quad (4)$$

where $\omega_d = \omega_c$. In even lower magnetic fields (the weak field regime) the stochastic sea spans almost the entire phase space. Correspondingly, the static conductivity is exponentially small, $\sigma_{xx}(0) \propto e^{- (\omega_d / \omega_c)^{3/2}}$ when $\omega_d \ll \omega_c$ (the strong and the intermediate field regimes), rapidly blows up near the $\omega_d = \omega_c$ point (the boundary of the intermediate and the weak field regimes), and finally crosses over to the Drude-Lorentz formula (1) in weak fields [8].

Our goal is to show that the dynamical conductivity also exhibits a rapid change near the $\omega_d = \omega_c$ point; the aforementioned collapse of the CR linewidth. This can be done on the basis of the classical formula for $\sigma_+(\omega)$,

$$\sigma_+(\omega) = \frac{\sigma_0}{\tau} \int_0^{\infty} dt \exp[i \omega t - i \theta(0)]. \quad (5)$$

Suppose that $\Delta \omega \equiv \omega - \omega_c$ is smaller than $\omega_c$ by absolute value, then the full equation of motion for $\theta$ can be replaced by its average over the cyclotron period,

$$\frac{d\theta}{dt} = \omega_c + \delta \omega, \quad \delta \omega = \frac{1}{m \omega_c R_c} \frac{\partial}{\partial R_c} U_0(\rho, R_c), \quad (6)$$

where $\delta \omega$ is the local correction to the gyration frequency of the velocity vector. Such a correction comes from an additional centripetal force exerted on the electron by the random potential.

In the strong and intermediate fields ($\omega_e \gg \omega_d$) most of the guiding centers are localized on the periodic orbits of small size $\ll d$. The correction $\delta \omega$ does not vary much on this length scale. To find $\Delta \omega_{1/2}$ up to a numerical factor, we can use an approximation $\delta \omega = \text{const}$. Substituting this into Eqs. (5) and (6), we find that $\Re \sigma_+(\omega)$ has the form of a Gaussian peak with width $\Delta \omega_{\text{rms}}$, and so

$$\Delta \omega_{1/2} \sim [\delta \omega]_{\text{rms}} = \text{const} \times \frac{W}{m \omega_c d^2}, \quad R_c \ll d, \quad (7)$$

The broadening of the CR line is of the inhomogeneous type, and comes from the places where $\Delta \omega = \delta \omega$ [9]. Equations (7) and (8) show that $\Delta \omega_{1/2}$ linearly increases as a function of $\omega_c^{-1}$ in the strong-field regime, reaches its maximum at $R_c \sim d$, and then decreases in the intermediate fields; see Fig. 1.

The situation is different in the weak fields where most of the trajectories are extended and ergodic. In this case the broadening is of the homogeneous type and is much smaller than $[\delta \omega]_{\text{rms}}$ because of the motional narrowing. The crossover between the two types of broadening occurs at the low field end of the intermediate-field regime.

According to Eqs. (3) and (8), $[\delta \omega]_{\text{rms}} \ll \omega_d$ in this regime. Because of this inequality, the orbits of low frequency $\Omega \ll \omega_d$ whose perimeter length is typically $(\omega_d / \Omega) d$ turn out to be important. To show this we present $\sigma_+(\omega)$ as follows,

$$\sigma_+(\omega) = \frac{\sigma_0}{\tau} \int_0^{\infty} d\Omega f(\Omega) I(\Omega, \Delta \omega), \quad (9)$$

$$I(\Omega, \Delta \omega) = \int_0^{\infty} dt \exp \left[ i \Delta \omega t - i \int_0^t dt' \delta \omega(t') \right] \Omega^{-1},$$

where $\langle \rangle_\Omega$ denotes the average over the orbits of frequency $\Omega$, and $f(\Omega)$ is the probability density of finding an electron with energy $E_F$ on such an orbit (with the convention that $\Omega = +0$ for the unbounded trajectories of the web). The correlations in $\delta \omega(\rho)$ decay sufficiently fast ($\sim \rho^{-3}$) with distance; hence, the high order correlators of $\delta \omega$ can be neglected, and we arrive at

$$I = \int_0^{\infty} \exp \left[ i \Delta \omega t - \frac{1}{2} \int_0^t dt_2 (\delta \omega(t_1) \delta \omega(t_2))_\Omega \right].$$
For the same reason we can approximate the second order correlator by the sum of isolated short pulses,
\[
\langle \delta \omega(t_1) \delta \omega(t_2) \rangle_{\Omega} = \sum_{n=-\infty}^{\infty} P(t_1 - t_2 - \frac{2\pi n}{\Omega}),
\]
where \( P(0) = \langle \delta \omega^2 \rangle \) and \( P(t) \ll P(0) \) for \( |t| \gg \omega_d^{-1} \).
In fact, if both \( \Omega \) and \( |\Delta \omega| \) are much smaller than \( \omega_d \),
the actual functional form of the pulses is unimportant,
and we can replace \( P(t) \) by \( 2\nu \delta(t) \), where
\[
\nu \sim \langle \delta \omega^2 \rangle/\omega_d \sim \omega_c/\omega_d \tau.
\]
After such approximations and some algebra we find:
\[
\text{Re} I = \frac{\pi}{\sqrt{2\Omega \nu}} \sum_{n=-\infty}^{\infty} c_n \exp \left[-\frac{\pi(\Delta \omega - n\Omega)^2}{2\Omega \nu} \right],
\] (12)
\[
c_n = \int_0^1 dx \cos(\pi nx) \exp \left[-\frac{\pi(2x - x^2)}{2\Omega \nu} \right].
\] (13)
If \( \Omega \gg \nu \), then \( c_0 = 1 \) and \( c_n = \nu/\pi n^2 \Omega, n \neq 0 \),
so that \( \text{Re} f(\Omega, \Delta \omega) \) as a function of \( \Delta \omega \) is the sum of
narrow Gaussians centered at points \( \Delta \omega = n\Omega \). In the opposite limit the Gaussians merge into a single Lorentzian,
\[
\text{Re} I(\Omega, \Delta \omega) = \frac{\nu}{(\Delta \omega)^2 + \nu^2}, \quad \Omega \ll \nu,
\] (14)
\[
|\Delta \omega| \ll \omega_d.
\]
There is a simple way to understand this equation. As discussed above, the random potential modifies the local
gyration frequency of the velocity vector by \( \delta \omega \). The sign
of the correction changes randomly each time the guiding
center travels the distance \( \sim d \). This causes the angular
diffusion of the velocity vector and therefore, the momentum
relaxation, which is superimposed on the regular precession
with frequency \( \omega_c \). The time \( \omega_d/(\delta \omega^2) = \nu^{-1} \)
plays the role of the effective relaxation time, and so
Eq. (14) is expected by analogy with Eq. (1).

To finish the calculation of \( \sigma\nu(\omega) \) we need to know \( f(\Omega) \). It is determined by the properties of the Fourier
transform \( \tilde{C}(q) \) of the correlator \( C(r) \equiv \langle U(0)U(r) \rangle \),
viz. by the exponent \( H \) in the asymptotic expression
\[
\int_{q} 2q dq q^2 \langle q R_c \rangle \tilde{C}(q) \propto q^{-2H},
\] (15)
where \( J_0 \) is the Bessel function. In a practically relevant
case of the potential created by randomly positioned donors
located in a narrow layer the distance \( d \) away from the
2DEG, \( H = -1 \) if \( q \ll R_c^{-1} \) and \( H = -1/2 \)
if \( R_c^{-1} \leq q \ll d^{-1} \). In both cases
\[
f(\Omega) \approx \Omega^{-1}(\Omega/\omega_d)^s, \quad \Omega_{\text{web}} \ll \omega_d, \quad \Omega_{\text{web}} \ll \omega_d, \quad \omega_d, \quad \omega_d.
\] (16a)
\[
= (\Omega_{\text{web}}/\omega_d)^s \delta(\Omega - 0) \quad \text{at other } \Omega, \quad \Omega_{\text{web}} \ll \omega_d.
\] (16b)
where \( s \) is very close to \( 1/2 \) [see Isichenko [11] for
details; \( \Omega \) \( f(\Omega) \) corresponds to his \( F(\Omega) \)]. The quantity
\( \Omega_{\text{web}} \sim \omega_d e^{-(\omega_d/\omega_d)^{s/1}} \) is the drift frequency of such
orbits within the stochastic web that give the dominant

We can now substitute Eqs. (12)–(16) into Eq. (9).
Since \( s < 1 \), only the \( n = 0 \) term in Eq. (12) is important
for \( \nu \ll \Omega \leq \omega_d \). Also, it is safe to assume that
\( \omega_d/\nu \ll e^{2/(2s-1)} \) [12]. With this in mind, we obtain the result
\[ \text{Re} \sigma\nu(\omega) = (\sigma\nu/\tau)[S_1(\omega) + S_2(\omega)], \]
where
\[
S_1(\omega) = \frac{\Omega_{\text{web}}}{\omega_d} \frac{1}{1 + (\Delta \omega/\nu)^2}, \quad |\Delta \omega| \leq \omega_d,
\]
\[
S_2(\omega) = \frac{\pi^2 \nu}{8 \omega_d} \ln \left[ \frac{\omega_d \nu}{\max\{ (\Delta \omega)^2, \Omega_{\text{web}} \}} \right].
\]
If \( \Omega_{\text{web}} \ll \nu \ll \omega_d \), then \( \Delta \omega_{1/2} \)
determined by \( S_2(\omega) \), and thus \( \Delta \omega_{1/2} \ll \sqrt{\omega_d \nu} \sim \sqrt{\omega_c/\tau} \), in accordance with Eq. (8). The CR line shape
for this case is depicted in Fig. 2(a).

Let us now discuss the aforementioned collapse of the
CR linewidth. \( \Omega_{\text{web}} \) grows exponentially as the magnetic
field decreases and eventually becomes larger than \( \nu \).
From this moment, the maximum value of \( S_2 \) rapidly decreases and that of \( S_1 \) does the opposite. It gives rise to
a narrow Lorentzian peak at the center of the CR line
(see Fig. 2(b)). At the \( \omega_d = \omega_c \) point where \( \Omega_{\text{web}} \sim \omega_d \), it is this
peak that determines \( \Delta \omega_{1/2} \). Thus, \( \Delta \omega_{1/2} \) drops to the
value of \( \nu = \omega_c/\omega_d \tau = \tau^{-1} \) [Fig. 2(c)]. From this
point on, i.e., in the weak-field regime, Eq. (1) applies.

Several comments are in order here. Formula (7) for the
strong-field regime is not new (see Refs. [1–3]). However,
its derivation has remained unsatisfactory: Ando’s
work [1] is based on the self-consistent Born approximation
(SCBA), which is invalid in this regime; Prasad and
Fujita [2] ignored the localized nature of the particle motion;
Bychkov and Iordanskii [3] de facto assumed that the
electron’s trajectory (in the absence of interactions) is
\( \delta \omega(\rho) = \text{const} \) instead of the correct \( U_0(\rho) = \text{const} \).

So far we have studied the case \( k F d \gg (E_F/W)^{2/3} \).
In the opposite limit an additional “quantum” region appears
on the phase diagram of Fig. 3 [10]. Its boundary is
formed by the lines \( l = d \) where \( l = \sqrt{\hbar/ma_c} \) is the
magnetic length, and \( \omega_c \tau_q = 1 \) where \( \tau_q = \tau/(k F d)^3 \)
is the quantum lifetime. The lower edge of this region, i.e.,
the line \( k F d = 1 \) was discussed in the beginning of this Letter.
Now we will show that the collapse of the CR

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FIG. 2. The CR line shape before (a), in the course (b), and immediately after (c) the collapse.
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linewidth occurs in the case $1 \ll k_F d \ll (E_F/W)^{2/3}$ as well, and that its position on the phase diagram is given by the line $l = d$ (which is the same as $R_c/d = k_F d$). Contrary to the “classical” case, the quantum collapse takes place when the Landau levels are well separated.

The CR line shape in the quantum region can be studied within the SCBA. Doing the integration in Eq. (2.12) of Ref. [1] for the case of $E_F$ positioned between the Landau levels and $|\Delta \omega| \ll \sqrt{\omega_c/\tau_q}$, we find that the CR line is roughly Lorentzian, while its median width is given by

$$\Delta \omega_{1/2} = 0.66 \tau^{-1} \sqrt{\omega_c \tau_q}.$$  \hspace{1cm} (17)

On the other hand, in the intermediate-field region $\Delta \omega_{1/2}$ is given by Eq. (8). Therefore, the crossing of the $l = d$ line causes the drop of $\Delta \omega_{1/2}$ by a large factor of $k_F d$ Fig. 4). The physics of this “quantum” collapse is quite similar to that of the classical one: the crossing of the line $l = d$ is accompanied by an explosive growth of the quantum localization length of the states near the Landau level center (see Ref. [10] for discussion). The states with sufficiently large localization length contribute to $\sigma_+(\omega)$ in the form similar to Eq. (1) but with $\tau$ replaced by the effective transport time. Since the density of states at the Landau level center is larger than the zero field density of states by a factor of $\sqrt{\omega_c/\tau_q}$ [1], this effective transport time is smaller than $\tau$ by the same factor in agreement with Eq. (17).

Until now we considered $W$ a fixed parameter of the theory. In a more realistic model (see above) $W$ is determined by the concentration $n_i$ of randomly positioned donors and the screening properties of the 2DEG. Away from the strong-field regime, the screening is very much the same as in zero field, and one obtains $W \sim E_F \sqrt{n_i/k_F d}$. Thus, the “quantum” case $k_F d \ll (E_F/W)^{2/3}$ is realized if $n_i$ is sufficiently low, $n_i \ll k_F/d$. On the other hand, in the strong-field regime (far from the collapse), the strength of the screening, $W$, and $\Delta \omega_{1/2}$ oscillate with the filling factor [13]. The comprehensive description of such oscillations is complicated by the nonlinear screening effects [14].

The nonmonotonic dependence of $\Delta \omega_{1/2}$ on the magnetic field with the maximum at $R_c \sim d$ has been observed in Ref. [15], in agreement with our theory. However, a decisive confirmation of the CR collapse requires further experiments.

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[7] More precisely, these are the level lines of some effective potential, which is very close to $U_0$; see M. M. Fogler, Phys. Rev. B (to be published).
[8] If the quantum localization effects can be neglected.
[9] The Gaussian line shape as an artifact of the $\delta \omega \equiv \text{const}$ approximation.