2. **Problem Set 1**

2. (a) Find a branch cut for function $f(z) = \arcsin(z)$ that runs along the real axis but does not include the interval $(-1, 1)$.
   
   **Hint:** Use the substitution $f(z) = -i \ln \phi(z)$.

2. (b) Let $f(\sin \Theta) = \Theta$ for $-\frac{\pi}{2} < \Theta < \frac{\pi}{2}$. Compute $f(i)$, $f(2 + i 0)$ and $f(2 - i 0)$. Separate the real and imaginary parts explicitly.

3. Consider $f(z) = \frac{i}{2} \ln(1 + z^2)$.

   (a) Which of the following branch-cut schemes makes $f$ single-valued:

   ![Branch-cut schemes](image)

   (b) Same question for $g(z) = \frac{i}{2} \ln\left(\frac{i + z}{i - z}\right)$. \(\text{ (Watch out!)}\)

   (c) Let $g(x + iy) \to \frac{\pi}{2}$, as $x \to +\infty$, $y \to 0$. Find $g(-0)$ for the branch-cut you selected in part (b). Give the most complete answer.

4. Given that $\text{Im} f(x+iy) = e^{-y} \sin x$ and that $f$ is analytic, find $f(z)$. Give the most general expression. \(\text{Hint: Cauchy-Riemann conditions may help.}\)

4. (a) $f(z)$ is defined by the series expansion:

   $$f(z) = -\frac{3}{z^3} - \frac{2}{z^2} - \frac{1}{z} + \sum_{n=0}^{\infty} \frac{1}{n!} z^n$$

   Find $\text{Res} f(z)$, $z = 0$. \(\text{ (This question is really simple.)}\)
(b) Same question for \( f(z) = \frac{1}{z} + \frac{1}{z^3} + \frac{1}{z^5} + \ldots = \sum_{n=0}^{\infty} \frac{1}{z^{2n+1}}. \) (This part is a bit tricky.)

(c) Discuss the relation of (a) and (b) to Laurent theorem / Laurent expansion.

(5) (a) Find all the solutions of the equation \( \cos z = i. \)

(b) Find the approximation to the solutions of \( \cos z = i + r, \) where \( r \) is a large real number (both for \( r > 0 \) and \( r < 0 \)).

(c) Sketch the curve defined by equation \( \text{Im}(\cos z) = i \) in the complex plane of \( z. \) Restrict your attention to the strip \( 0 < \text{Re} z < 2\pi, \) i.e., one period.