1. \[ e \Delta \phi = k_B T \ln \frac{N_A N_D}{n_i^2} = 2 k_B T \ln \frac{N_D}{n_i} \]

\[ d_n = d_p = \sqrt{\frac{K}{2 \pi e^2} \frac{\Delta \phi}{2 N_0}} = \sqrt{\frac{K}{2 \pi e^2} \frac{n_i^2}{N_D}} \]

\[ = \left[ \frac{16.2 \cdot 1.38 \cdot 10^{-16} \cdot 300}{2 \cdot \pi \cdot (6 \cdot 10^{12})^2 \cdot 10^{17}} \ln \left( \frac{10^{17}}{24 \cdot 10^{13}} \right) \right]^{1/2} \text{cm} \]

\[ = 6.2 \cdot 10^{-6} \text{ cm} = 62 \text{ nm}. \]

2. \( d_n \) given by the same formula as in p-n junction in terms of \( \Delta \phi = \phi(\infty) - \phi(-\infty) \), if \( N_A \rightarrow \infty \).

To find \( \Delta \phi \) use the constancy of the electrochem. pot.

\( \mu = -e \phi(\infty) + \mu_{ch} = -e \phi(-\infty) - W = \text{const} \)

where we took the vacuum as the energy reference point, \( W \) is the metal's work function, and \( \mu_{ch} = E_c + k_B T \ln \frac{N_D}{N_e} \) is the chemical potential of the conduction-band electrons in Ge. With our convention for the energy reference, \( E_c = -\chi_e \), where \( \chi_e \) = electron affinity of Ge. Assembling all formulas together, we get \( e \Delta \phi = W - \chi_e + k_B T \ln \frac{N_D}{N_e} \).

To estimate \( N_e \) use the formula \( n_i^2 = N_e N_h e^{-E_g/2k_B T} \) [see Eq. (8.43) in Kittel], where we assume \( N_e \approx N_h \), so that \( N_e \approx n_i e^{E_g/2k_B T} \).

\[ e \Delta \phi = W - \chi_e - \frac{E_g}{2} + k_B T \ln \frac{N_D}{n_i} \]

where \( E_g = 0.67 \text{ eV} \) is the Ge gap.

\[ e \Delta \phi = \left[ 4.7 - 4 - \frac{0.67}{2} + \frac{300 K}{116.16 K} \ln \left( \frac{10^{17}}{2.4 \cdot 10^{13}} \right) \right] \text{eV} = 0.58 \text{ eV}. \]

\[ d_n = \sqrt{\frac{K (e \Delta \phi)}{2 \pi e^2} \frac{N_A / N_D}{N_A + N_D}} \bigg|_{N_A = \infty} \approx \sqrt{\frac{K}{2 \pi e^2} \frac{e \Delta \phi}{N_D}} \approx \left[ \frac{16.2 \cdot (1.6 \cdot 10^{-19} \cdot 0.58)}{2 \cdot (6 \cdot 10^{12})^2 \cdot 10^{17}} \right]^{1/2} \text{cm} \approx 100 \text{ nm}. \]
In this case the sign of the band curvature is opposite, and instead of a depletion layer, there forms an accumulation layer populated by conduction band electrons, see figure. This is known as population inversion. The amount of the band curvature is given by

\[ e\phi = W - \chi_G - \frac{E_g}{2} - k_B T \ln \left( \frac{N_A}{n_c} \right) \approx (2 - 4 - \frac{0.67}{2} - 0.21) \text{ eV} \approx -2.54 \text{ eV}. \]

\[
G = 800 \text{ J}^{-1} \text{ cm}^{-1}
\]

\[
1 \text{ J}^{-1} = 9 \times 10^{14} \text{ cm/s} \quad \Rightarrow \quad G = 7.2 \times 10^{14} \text{ s}^{-1}.
\]

"Magic conversion formula"